



AMENDMENTS TO THE CLAIMS

Claims 1-18 (Canceled)

19. (New) A method of deriving a set of closed time average turbulent moment equations for anisotropic turbulent flow in an anisotropic fluid comprising:

providing input to a general purpose computer defining a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

instructing the general purpose computer to calculate n^{th} order, wherein n is odd, directional kinetic energy fluxes and structure correlation equations closure relationships using $(n + 1)^{\text{th}}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Turbulent Energy

$$\begin{aligned} & \frac{\partial}{\partial t} [\overline{u_1'^2}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u_1'^2}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u_1'^2}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u_1'^2}] \\ & + 2 \left[\overline{u_1'^2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_1' u_2'} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1' u_3'} \frac{\partial \overline{u_1}}{\partial x_3} \right] \\ & + \frac{1}{\overline{\rho}} \left[\frac{\partial}{\partial x_1} [\overline{\rho u_1' u_1'^2}] + \frac{\partial}{\partial x_2} [\overline{\rho u_2' u_1'^2}] + \frac{\partial}{\partial x_3} [\overline{\rho u_3' u_1'^2}] \right] \\ & = 0 \end{aligned}$$

Turbulent Shear



$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'_2}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \\
& + \overline{u'_1 u'_2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'^2_2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u'_2 u'_3} \frac{\partial \overline{u_1}}{\partial x_3} \\
& + \overline{u'^2_1} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u'_1 u'_2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_1 u'_3} \frac{\partial \overline{u_2}}{\partial x_3} \\
& + \frac{1}{\overline{\rho}} \left[\frac{\partial}{\partial x_1} [\overline{\rho u'_2 u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{\rho u'_1 u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{\rho u'_1 u'_2 u'_3}] \right] \\
& = 0
\end{aligned}$$

Directional Turbulent Energy Fluxes

$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'^2_1}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'^2_1}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'^2_1}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'^2_1}] \\
& + 3 \left[\overline{u'_1 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'_2 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u'_3 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_3} \right] \\
& - 3 \overline{u'^2_1} \left[\frac{\partial}{\partial x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + 3 \overline{c^2_1} \left[\frac{\partial}{\partial x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + 3 \left[\overline{u'^2_1} \frac{\partial}{\partial x_1} [\overline{c^2_1}] + \overline{u'_1 u'_2} \frac{\partial}{\partial x_2} [\overline{c^2_1}] + \overline{u'_1 u'_3} \frac{\partial}{\partial x_3} [\overline{c^2_1}] \right] \\
& + \frac{\partial}{\partial x_1} [\overline{u'^2_1 u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2 u'^2_1}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3 u'^2_1}] \\
& + \left[\overline{u'^2_1 u'^2_1} - 3 \overline{u'^2_1} [\overline{u'^2_1}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_1} \\
& + \left[\overline{u'_1 u'_2 u'^2_1} - 3 \overline{u'_1 u'_2} [\overline{u'^2_1}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_2} \\
& + \left[\overline{u'_1 u'_3 u'^2_1} - 3 \overline{u'_1 u'_3} [\overline{u'^2_1}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_3} \\
& = 0
\end{aligned}$$

Directional Turbulent Energy Fluxes

$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'^2_2}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'^2_2}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'^2_2}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'^2_2}] \\
& + 2 \left[\overline{u'_1 u'^2_2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_2 u'^2_1} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_2}}{\partial x_3} \right] \\
& + \overline{u'_2 u'^2_2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u'_1 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'_3 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_3} \\
& - 2 \overline{u'_1 u'_2} \left[\frac{\partial}{\partial x_1} [\overline{u'^2_2}] + \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_2 u'_3}] \right] \\
& - \overline{u'^2_2} \left[\frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + 2 \left[\overline{c^2_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{c_1 c_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \right] \\
& + \overline{c_1 c_2} \frac{\partial}{\partial x_2} [\overline{u'^2_2}] + \overline{c^2_1} \frac{\partial}{\partial x_1} [\overline{u'^2_2}] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} [\overline{u'^2_2}] \\
& + 2 \left[\overline{u'^2_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \overline{u'_1 u'_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{u'_2 u'_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \right] \\
& + \overline{u'_1 u'_2} \frac{\partial}{\partial x_2} [\overline{c^2_2}] + \overline{u'^2_1} \frac{\partial}{\partial x_1} [\overline{c^2_2}] + \overline{u'_1 u'_3} \frac{\partial}{\partial x_3} [\overline{c^2_1}] \\
& + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2 u'^2_2}] + \frac{\partial}{\partial x_1} [\overline{u'^2_1 u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3 u'^2_2}] \\
& + \left[\overline{u'_1 u'_2 u'^2_2} - 3 \overline{u'_1 u'_2} [\overline{u'^2_2}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_2} \\
& + \left[\overline{u'^2_1 u'^2_2} - \overline{u'^2_1} [\overline{u'^2_2}] - 2 \overline{u'_1 u'_2} [\overline{u'_1 u'_2}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_1} \\
& + \left[\overline{u'_1 u'_3 u'^2_2} - \overline{u'_1 u'_3} [\overline{u'^2_2}] - 2 \overline{u'_1 u'_2} [\overline{u'_2 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_3} \\
& = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'_2 u'_3}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2 u'_3}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2 u'_3}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2 u'_3}] \\
& + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'_3 u'^2_2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_2 u'^2_3} \frac{\partial \overline{u_1}}{\partial x_3} \\
& + \overline{u'_3 u'^2_1} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_1 u'^2_3} \frac{\partial \overline{u_2}}{\partial x_3} \\
& + \overline{u'_2 u'^2_1} \frac{\partial \overline{u_3}}{\partial x_1} + \overline{u'_1 u'^2_2} \frac{\partial \overline{u_2}}{\partial x_3} + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_3}}{\partial x_3} \\
& - \overline{u'_1 u'_2} \left[\frac{\partial}{\partial x_1} [\overline{u'_1 u'_3}] + \frac{\partial}{\partial x_2} [\overline{u'_2 u'_3}] + \frac{\partial}{\partial x_3} [\overline{u'^2_3}] \right] \\
& - \overline{u'_1 u'_3} \left[\frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_2} [\overline{u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{u'_2 u'_3}] \right] \\
& - \overline{u'_2 u'_3} \left[\frac{\partial}{\partial x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + \overline{u'^2_1} \frac{\partial}{\partial x_1} [\overline{c_2 c_3}] + \overline{u'_1 u'_2} \frac{\partial}{\partial x_2} [\overline{c_2 c_3}] + \overline{u'_1 u'_3} \frac{\partial}{\partial x_3} [\overline{c_2 c_3}] \\
& + \overline{u'_1 u'_2} \frac{\partial}{\partial x_1} [\overline{c_1 c_3}] + \overline{u'^2_2} \frac{\partial}{\partial x_2} [\overline{c_1 c_3}] + \overline{u'_2 u'_3} \frac{\partial}{\partial x_3} [\overline{c_1 c_3}] \\
& + \overline{u'_1 u'_3} \frac{\partial}{\partial x_1} [\overline{c_1 c_2}] + \overline{u'_2 u'_3} \frac{\partial}{\partial x_2} [\overline{c_1 c_2}] + \overline{u'^2_3} \frac{\partial}{\partial x_3} [\overline{c_1 c_2}] \\
& + \overline{c^2_1} \frac{\partial}{\partial x_1} [\overline{u'_2 u'_3}] + \overline{c_1 c_2} \frac{\partial}{\partial x_2} [\overline{u'_2 u'_3}] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} [\overline{u'_2 u'_3}] \\
& + \overline{c_1 c_3} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{c_2 c_3} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \overline{c^2_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \\
& + \overline{c_1 c_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_3}] + \overline{c^2_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_3}] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \\
& + \frac{\partial}{\partial x_1} [\overline{u'_2 u'_3 u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_3 u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2 u'^2_3}] \\
& + \left[\overline{u'_2 u'_3 u'^2_1} - \overline{u'_2 u'_3} [\overline{u'^2_1}] - 2 \overline{u'_1 u'_2} [\overline{u'_1 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_1} \\
& + \left[\overline{u'_1 u'_3 u'^2_2} - \overline{u'_1 u'_3} [\overline{u'^2_2}] - 2 \overline{u'_1 u'_2} [\overline{u'_2 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_2} \\
& + \left[\overline{u'_1 u'_2 u'^2_3} - \overline{u'_1 u'_2} [\overline{u'^2_3}] - 2 \overline{u'_1 u'_3} [\overline{u'_2 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_3} \\
& = 0
\end{aligned}$$

20. (New) A computer readable storage medium containing a set of instructions for a general purpose computer, the set of instructions defining a method of deriving a set of closed time average turbulent moment equations for anisotropic turbulent flow in an anisotropic fluid comprising:

defining a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

calculating n^{th} order , wherein n is odd, directional kinetic energy fluxes and structure correlation equations closure relationships using $(n + 1)^{\text{th}}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Turbulent Energy

$$\begin{aligned} & \frac{\partial}{\partial t} [\overline{u_1'^2}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u_1'^2}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u_1'^2}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u_1'^2}] \\ & + 2 \left[\overline{u_1'^2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_1' u_2'} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1' u_3'} \frac{\partial \overline{u_1}}{\partial x_3} \right] \\ & + \frac{1}{\rho} \left[\frac{\partial}{\partial x_1} [\overline{\rho u_1' u_1'^2}] + \frac{\partial}{\partial x_2} [\overline{\rho u_2' u_1'^2}] + \frac{\partial}{\partial x_3} [\overline{\rho u_3' u_1'^2}] \right] \\ & = 0 \end{aligned}$$

Turbulent Shear

$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'_2}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \\
& + \overline{u'_1 u'_2} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'^2_2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u'_2 u'_3} \frac{\partial \overline{u_1}}{\partial x_3} \\
& + \overline{u'^2_1} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u'_1 u'_2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_1 u'_3} \frac{\partial \overline{u_2}}{\partial x_3} \\
& + \frac{1}{\overline{\rho}} \left[\frac{\partial}{\partial x_1} [\overline{\rho u'_2 u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{\rho u'_1 u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{\rho u'_1 u'_2 u'_3}] \right] \\
& = 0
\end{aligned}$$

Directional Turbulent Energy Fluxes

$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'^2_1}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'^2_1}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'^2_1}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'^2_1}] \\
& + 3 \left[\overline{u'_1 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'_2 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u'_3 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_3} \right] \\
& - 3 \overline{u'^2_1} \left[\frac{\partial}{\partial x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + 3 \overline{c^2_1} \left[\frac{\delta}{\delta x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + 3 \left[\overline{u'^2_1} \frac{\partial}{\partial x_1} [\overline{c^2_1}] + \overline{u'_1 u'_2} \frac{\partial}{\partial x_2} [\overline{c^2_1}] + \overline{u'_1 u'_3} \frac{\partial}{\partial x_3} [\overline{c^2_1}] \right] \\
& + \frac{\partial}{\partial x_1} [\overline{u'^2_1 u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2 u'^2_1}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3 u'^2_1}] \\
& + \left[\overline{u'^2_1 u'^2_1} - 3 \overline{u'^2_1} [\overline{u'^2_1}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_1} \\
& + \left[\overline{u'_1 u'_2 u'^2_1} - 3 \overline{u'_1 u'_2} [\overline{u'^2_1}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_2} \\
& + \left[\overline{u'_1 u'_3 u'^2_1} - 3 \overline{u'_1 u'_3} [\overline{u'^2_1}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_3} \\
& = 0
\end{aligned}$$

Directional Turbulent Energy Fluxes

$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'^2_2}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'^2_2}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'^2_2}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'^2_2}] \\
& + 2 \left[\overline{u'_1 u'^2_2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_2 u'^2_1} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_2}}{\partial x_3} \right] \\
& + \overline{u'_2 u'^2_2} \frac{\partial \overline{u_1}}{\partial x_2} + \overline{u'_1 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'_3 u'^2_1} \frac{\partial \overline{u_1}}{\partial x_3} \\
& - 2 \overline{u'_1 u'_2} \left[\frac{\partial}{\partial x_1} [\overline{u'^2_2}] + \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_2 u'_3}] \right] \\
& - \overline{u'^2_2} \left[\frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + 2 \left[\overline{c^2_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{c_1 c_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \right] \\
& + \overline{c_1 c_2} \frac{\partial}{\partial x_2} [\overline{u'^2_2}] + \overline{c^2_1} \frac{\partial}{\partial x_1} [\overline{u'^2_2}] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} [\overline{u'^2_2}] \\
& + 2 \left[\overline{u'^2_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \overline{u'_1 u'_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{u'_2 u'_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \right] \\
& + \overline{u'_1 u'_2} \frac{\partial}{\partial x_2} [\overline{c^2_2}] + \overline{u'^2_1} \frac{\partial}{\partial x_1} [\overline{c^2_2}] + \overline{u'_1 u'_3} \frac{\partial}{\partial x_3} [\overline{c^2_1}] \\
& + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2 u'^2_2}] + \frac{\partial}{\partial x_1} [\overline{u'^2_1 u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3 u'^2_2}] \\
& + \left[\overline{u'_1 u'_2 u'^2_2} - 3 \overline{u'_1 u'_2} [\overline{u'^2_2}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_2} \\
& + \left[\overline{u'^2_1 u'^2_2} - \overline{u'^2_1} [\overline{u'^2_2}] - 2 \overline{u'_1 u'_2} [\overline{u'_1 u'_2}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_1} \\
& + \left[\overline{u'_1 u'_3 u'^2_2} - \overline{u'_1 u'_3} [\overline{u'^2_2}] - 2 \overline{u'_1 u'_2} [\overline{u'_2 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_3} \\
& = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial t} [\overline{u'_1 u'_2 u'_3}] + \overline{u_1} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2 u'_3}] + \overline{u_2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2 u'_3}] + \overline{u_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2 u'_3}] \\
& + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_1}}{\partial x_1} + \overline{u'_3 u'^2_2} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_2 u'^2_3} \frac{\partial \overline{u_1}}{\partial x_3} \\
& + \overline{u'_3 u'^2_1} \frac{\partial \overline{u_2}}{\partial x_1} + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_2}}{\partial x_2} + \overline{u'_1 u'^2_3} \frac{\partial \overline{u_2}}{\partial x_3} \\
& + \overline{u'_2 u'^2_1} \frac{\partial \overline{u_3}}{\partial x_1} + \overline{u'_1 u'^2_2} \frac{\partial \overline{u_2}}{\partial x_3} + \overline{u'_1 u'_2 u'_3} \frac{\partial \overline{u_3}}{\partial x_3} \\
& - \overline{u'_1 u'_2} \left[\frac{\partial}{\partial x_1} [\overline{u'_1 u'_3}] + \frac{\partial}{\partial x_2} [\overline{u'_2 u'_3}] + \frac{\partial}{\partial x_3} [\overline{u'^2_3}] \right] \\
& - \overline{u'_1 u'_3} \left[\frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_2} [\overline{u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{u'_2 u'_3}] \right] \\
& - \overline{u'_2 u'_3} \left[\frac{\partial}{\partial x_1} [\overline{u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \right] \\
& + \overline{u'^2_1} \frac{\partial}{\partial x_1} [\overline{c_2 c_3}] + \overline{u'_1 u'_2} \frac{\partial}{\partial x_2} [\overline{c_2 c_3}] + \overline{u'_1 u'_3} \frac{\partial}{\partial x_3} [\overline{c_2 c_3}] \\
& + \overline{u'_1 u'_2} \frac{\partial}{\partial x_1} [\overline{c_1 c_3}] + \overline{u'^2_2} \frac{\partial}{\partial x_2} [\overline{c_1 c_3}] + \overline{u'_2 u'_3} \frac{\partial}{\partial x_3} [\overline{c_1 c_3}] \\
& + \overline{u'_1 u'_3} \frac{\partial}{\partial x_1} [\overline{c_1 c_2}] + \overline{u'_2 u'_3} \frac{\partial}{\partial x_2} [\overline{c_1 c_2}] + \overline{u'^2_3} \frac{\partial}{\partial x_3} [\overline{c_1 c_2}] \\
& + \overline{c_1^2} \frac{\partial}{\partial x_1} [\overline{u'_2 u'_3}] + \overline{c_1 c_2} \frac{\partial}{\partial x_2} [\overline{u'_2 u'_3}] + \overline{c_1 c_3} \frac{\partial}{\partial x_3} [\overline{u'_2 u'_3}] \\
& + \overline{c_1 c_3} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_2}] + \overline{c_2 c_3} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_2}] + \overline{c_3^2} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2}] \\
& + \overline{c_1 c_2} \frac{\partial}{\partial x_1} [\overline{u'_1 u'_3}] + \overline{c_2^2} \frac{\partial}{\partial x_2} [\overline{u'_1 u'_3}] + \overline{c_2 c_3} \frac{\partial}{\partial x_3} [\overline{u'_1 u'_3}] \\
& + \frac{\partial}{\partial x_1} [\overline{u'_2 u'_3 u'^2_1}] + \frac{\partial}{\partial x_2} [\overline{u'_1 u'_3 u'^2_2}] + \frac{\partial}{\partial x_3} [\overline{u'_1 u'_2 u'^2_3}] \\
& + \left[\overline{u'_2 u'_3 u'^2_1} - \overline{u'_2 u'_3} [\overline{u'^2_1}] - 2 \overline{u'_1 u'_2} [\overline{u'_1 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_1} \\
& + \left[\overline{u'_1 u'_3 u'^2_2} - \overline{u'_1 u'_3} [\overline{u'^2_2}] - 2 \overline{u'_1 u'_2} [\overline{u'_2 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_2} \\
& + \left[\overline{u'_1 u'_2 u'^2_3} - \overline{u'_1 u'_2} [\overline{u'^2_3}] - 2 \overline{u'_1 u'_3} [\overline{u'_2 u'_3}] \right] \frac{1}{\overline{\rho}} \frac{\partial \overline{\rho}}{\partial x_3} \\
& = 0
\end{aligned}$$